Decision Analysis in Information Systems

II – Linear Programming

Introduction

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Section: Linear Programming

- **Introduction**
- Geometrical Perspective
- Algorithmic Perspective
- Transportation Problem
- Implementation in R

Linear Programming - Literature

- Bertsimas, D., & Freund, R.M. (2004). Data, Models, and Decisions – The Fundamentals of Management Science. Dynamic Ideas, Belmont, MA, USA.

- Hillier, F.S., & Lieberman, G.J. (2010). Introduction to Operations Research. Ninth Edition, International Edition 2010, McGraw-Hill, New York, NY, USA.

- Zimmermann, H.J. (2008). Operations Research – Methoden und Modelle. Second Edition, Vieweg, Wiesbaden, Germany.

Examples of application areas

- **Production planning:** Given several products with varying production requirements and cost structures, determine how much of each product to manufacture in order to maximize profits.
- **Scheduling:** Given a certain workforce, determine an optimal work schedule that maximizes worker preferences while adhering to scheduling rules.
- **Network installation:** Given point-to-point demands on a network, install capacities on the telecom links so as to minimize installation and routing costs.
- **Transportation problem:** Given a set of sources and a set of destinations. Find the lowest transportation costs from the sources to the destinations subject to fixed demand and capacity constraints.

Terminology (1)

- Decision Variable
  - Describes a decision that needs to be made
  - E.g. “How many items to produce?”
- Objective function
  - Expression that needs to be minimized or maximized
  - Expression in terms of the variables
  - E.g. “Profit maximization”
- Constraint
  - Expression that restricts the values of the variables
  - E.g. “Constraint on total working hours per week”

Terminology (2)

- Feasible plan / feasible solution
  - Assignment of decision variables satisfying all constraints and non-negativity conditions
- Goal
  - Find a feasible plan that optimizes the objective function
  - Assignment of decision variables that satisfies all constraints
- Optimal plan
  - Feasible plan that achieves the best value of the objective function over all other feasible plans

Linear Optimization Problem - Definition

A problem is called a linear optimization problem if

- all constraints (“≤”, “≥”, “=”) are linear functions and
- the objective function is linear.

e.g.,	$A + 2B - 5C + 7D + 174$	(linear function)
	$A\,B + 7\,C + 8\,D + 32$	(not linear)
	$4\,A + 5 \sin(B)$	(not linear)
	$A^2 + 4\,B - 2\,C + 5\,D$	(not linear)

Steps to construct a linear optimization model

1. Define the decision variables.
2. Construct the objective function in terms of the decision variables.
3. Construct the constraints in terms of the decision variables.
4. Construct the non-negativity conditions.

A “Partial Taxonomy” of Optimization Problems

Linear Optimization (LP)
objective and constraints are linear expressions

Integer Linear Optimization (IP)
variables are restricted to discrete (integer) values

Mixed - Integer Linear Optimization (MIP)
some variables are continuous, some are discrete

Nonlinear Optimization (NLP)
objective and/or constraints are non-linear expressions